LOCAL DISCONTINUOUS GALERKIN FORMULATIONS FOR BEAM PROBLEMS[1]

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Standard, displacement finite element formulation of many structural mechanics problems is marred by the phenomenon called locking, either shear or membrane or both. The nature of the phenomenon is well understood and, in general terms, has to do with the lack of appropriate matching (consistency) between functions used to approximate various components of the displacement field. As a result, the pattern of structural deformation, allowed by the finite element approximation, includes unrealistically high contribution of shear deformation and/or membrane deformation. In structures, these are highly energy absorbing modes of deformation and their excessive presence in the model leads to locking.

Over the last twenty years or so effective remedies to locking have been found for many finite element formulations of structural problems. They include several different approaches, of which the reduced integration technique, mixed methods and assumed strain methods are probably the best known representatives. While different at the first glance, many of those approaches can be formally shown to be equivalent. This is quite natural given the fact that they all are designed to reduce the unduly high contribution of the shear and/or membrane energy within each finite element of the mesh. Despite the existence of an array of different approaches designed to eliminate locking in finite element analysis of structures, different structural problems enjoyed different degree of success. It appears that one-dimensional structures, such as beams and arches (both two- and three-dimensional), can be successfully solved using elements of various order and any of the existing methods, both within linear and nonlinear range. Linear analysis of plate problems can also be successfully conducted using a variety of elements and methods. However, when shells (or plates undergoing large deformations) are involved, the situation is much more tenuous. The successful approaches in this case are highly specialized and they are not as general as in the case of other structures.

The ultimate goal of the research presented here is improvement of the current state-of-the-art in linear and nonlinear analysis of shells using the Local Discontinuous Galerkin (LDG) Method. This approach has been selected considering the fact that here, in addition to deformation of the element itself, one can introduce additional flexibility by assigning desirable properties to the element interfaces (jump properties). Given the difficulties with capturing the correct behavior of shells only by appropriate formulation of shell elements, this feature of the LDG approach seems attractive from the point of view of elimination of locking and deserves to be explored. In this presentation we first provide various versions of the LDG formulation of the beam problems (Timoshenko/Bernoulli-Euler) as a model problem, (to subsequently explore extensions to shells). This choice of the problem allows us to make extensive comparisons with the existing effective approaches. Our analysis includes criteria of stability of LDG formulations as well as the conditions for existence and uniqueness of the solutions. We then examine the ability of various formulations to eliminate locking, then we explore the role different parameters play in this process. We provide numerical examples illustrating our theoretical predictions and those emerging from the physical interpretation of the problem.

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